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Chapter 1

Analytical Techniques for Obtaining Current and Concentration in Enzymatic Biofuel Cells for Various Enzymatic Scheme

K Saranya¹; Marwan Abukhaled²; V Mohan¹; L Rajendran^{3}*

¹*Department of Mathematics, Thiagarajar College of Engineering, Madurai-625 015, Tamil Nadu, India.*

²*Department of Mathematics and Statistics, American University of Sharjah, United Arab Emirates.*

³*Department of Mathematics, Academy of Maritime Education and Training (AMET), Deemed to be University, Chennai-603112, India.*

**Correspondence to: L Rajendran, Department of Mathematics, Academy of Maritime Education and Training (AMET), Deemed to be University, Chennai-603112, India.*

Email: raj_sms@rediffmail.com

Abstract

In this chapter, mathematical models for an enzymatic biofuel cell for the steady and transient states are discussed. Detailed maps and charge balances are combined with a model for the reaction mechanism in the electrodes. It is important that theoretical models are developed to reduce the burden on laboratory-based testing and accelerate the development of practical systems in enzymatic biofuel cells. In this study, recent key developments in the modeling of biofuel cells and bio-fuel cell technology are reviewed. Enzymatic biofuel cells can be modeled with linear and nonlinear partial differential equations (PDEs) of substrates and products. This chapter is dedicated to the modeling of enzymatic biofuel cells by analytical and numerical solutions of governing PDEs.

Keywords: Mathematical Modeling; Enzymatic Biofuel Cells; Nonlinear Equations; Analytical Solution; Numerical Solution.

1. Introduction

Biofuel cells operate on the same principles as conventional fuel cells where chemical energy is converted into electrical energy. It is true that biofuel cells have significantly lower power outputs and shorter lifetime compared to conventional cells. But they have some highly attractive properties such as their use of renewable bio-catalysts, ability to operate on a much broader range of fuels, non-polluting, flexibility of fuels and mild operating conditions. Enzymatic fuel cells (EFCs) employ enzymes in isolated and purified forms, which enable the construction of relatively well-defined systems that exhibit a high specificity toward the desired reactions, and avoids the use of dangerous micro-organisms. Biofuel cells have received significant attention in the last few decades due to medical applications like its ability in powering bioelectronics and portable electronic devices such as cardiac pacemakers.

Mathematical modeling and numerical simulation could play vital roles in further understanding and developing enzymatic biofuel cells. Carefully validated models can be used in conjunction with laboratory studies to investigate the reaction environment and design of electrodes for the development of practical systems. Thus far, a small number of mathematical models have been developed and with few notable exceptions, these models are highly simplified, neglecting crucial features such as the dynamics, spatial non-uniformities, ion migration, fluid flow, and heat transport [1].

In this chapter, detailed mass and charge balances combined with models for the multistep reaction mechanisms in the electrodes are discussed. The obtained results (analytical and numerical) are rigorously fitted to half-cell experimental data and compared against whole cell data, demonstrating excellent agreement.

2. Nonlinear Equation in Enzymatic Biofuel Cell

The action of the enzyme in a biofuel cell can be modeled with a nonlinear partial differential equation with appropriate boundary conditions. This chapter is devoted to the modeling of biofuel cells by providing analytical solution to systems of nonlinear ODEs or PDEs. Some of the analytical solutions of the strongly nonlinear systems in biofuel cells with various enzyme reaction mechanisms are given in **Table 1**. The modeling of biofuel cell at steady state and internal diffusion limitation is considered with special contributions of various enzyme reaction scheme. The non-stationary response of amperometric such as volumetric or normal pulse voltammetry in biofuel cells is analyzed. Most mathematical models are described by a nonstationary reaction-diffusion equation, but special emphasis is placed on mathematical models of enzymatic biofuel cells at steady state and non-steady state conditions for various reaction schemes because it can be used to produce renewable electricity, thermal energy or transportation fuels (biofuels). Kirthiga et al. [2] derived a closed-form analytical expression pertaining to the concentration of substrate, biomass, and ethanol in terms of all

kinetic parameters for anaerobic solid-state fermentation in biofuel production. Mediated bioelectrocatalysis, which is very much useful to build bioreactors, biofuel cells, and biosensor can also be employed for measuring enzyme kinetics parameters. Rajendran et al. [3] discussed the mathematical modeling and numerical simulation of a mediated bioelectrocatalysis. Analytical expressions of the concentration and current of mediated bioelectrocatalysis for the steady and non-steady conditions have been derived using Danckwerts' expression and the homotopy perturbation method.

Praveen et al. [4] discussed the modeling of immobilized enzymes under different kinetics mechanism viz. simple Michaelis–Menten, uncompetitive substrate inhibition, total competitive product inhibition, total non-competitive product inhibition and reversible Michaelis–Menten reaction. All these models are based on nonlinear reaction-diffusion equations containing nonlinear terms related to Michaelis–Menten kinetics of the enzymatic reaction. Analytical expressions for the concentration of co-substrate for steady and transient conditions using the homotopy perturbation method (HPM) were derived by Rasi et al. [5]. Analytical expressions of the plateau current were also analyzed for steady and transient conditions.

The current–potential curve for redox polymer mediated kinetic scheme pertaining to biofuel cells was analyzed by Saravana Kumar et al. [6]. An approach of the HPM was employed to analytically solve the ping-pong reaction scheme and hence estimate the current density. The validity of the theoretical approach was demonstrated using the known previous experimental data for a series of osmium based redox polymers, involving oxygen as the substrate with laccase being the enzyme for biocathode fuel cell reactions. Two novel graphical procedures for estimating the Michaelis–Menten constants and catalytic rate constants from the experimental current densities were also reported.

Praveen et al. [7] presented theoretical analysis of a nonlinear reaction-diffusion process involved in packed bed photobioreactor with immobilized-cell. Modified Adomian decomposition method was used to derive the concentration under steady-state condition. The obtained analytical results were shown to be in good agreement with simulated results for all different enzyme concentrations with simulated results. Saravana Kumar et al. [8] discussed a mathematical model for reaction-diffusion processes in a biofuel cell electrode. The model was based on a nonlinear system of reaction-diffusion equations containing a nonlinear term related to the rate of the enzyme reaction. Approximate closed form analytical expressions for the steady and non-steady state current densities at the electrode surface were obtained by employing a new approach of the homotopy perturbation method. A novel graphical procedure for estimating the Michaelis-Menten constants and turnover rate solely from the current-potential curve was suggested. Effect of the various controllable kinetic parameters such as diffusion of the mediator, Michaelis-Menten constant for substrate, the second-order rate constant, the thickness of the film, turnover rate and initial substrate concentration on the current density were discussed.

Saravana Kumar and Rajendran [9] discussed a mathematical model for the enzyme-entrapped conducting polymer modified electrode. Approximate expressions for the steady-state concentration of the various species in the film and the current response of the enzyme electrode, using the homotopy analysis method were derived. The redox enzyme in conducting polymer electrode being the most fundamental requirement for the development of amperometric biosensor, biofuel cell elements was discussed. Rasi and Rajendran [10] discussed a mathematical model for describing the oxidation of glucose in a multiscale porous biocatalytic electrode. The considered model was composed of two nonlinear differential equations accounting for reaction and diffusion within the hydrogel film. Adomian decomposition method was employed to derive approximate analytical expressions for the concentration of mediator, substrate, and current. The approximate solutions were found to be reliable when compared to MATLAB numerical solutions. Sensitivity analysis of the parameters were also discussed.

The transient current–potential response of the enzyme-catalyzed oxygen reduction reaction in biofuel cells was analyzed in [11]. The one-dimensional system of nonlinear reaction-diffusion equation was solved analytically using the homotopy method to derive the substrate concentrations and current densities pertaining to *chronoamperometry*. The response time required to obtain the steady state was investigated and the influence of film thickness, diffusion coefficients, and enzyme characteristics was discussed. The analytical equations were shown to be valid for experimental data on mediated electron transfer reactions of redox polymers containing osmium complexes.

Kirthiga and Rajendran [12] discussed the case where the enzyme reacts with an electroinactive substrate to produce an electroactive product, which is quickly oxidized or reduced at the electrode/film interface. Approximate analytical expression for concentrations of substrate, product and corresponding current response in the case of an enzyme immobilized system were derived using a modified approach of the HPM. The analytical results show excellent agreement with MATLAB numerical simulations. A novel graphical procedure for estimating the kinetic parameters and sensitivity analysis of the parameters from current density is also reported.

Recently an analytical method to solve the nonlinear differential equations in an immobilized enzyme film was presented [13]. Analytical expressions for concentrations of substrate and product have been derived for all values of dimensionless parameters. Dimensionless numbers that can be used to study the effects of enzyme loading, enzymatic gel thickness, and oxidation/ reduction kinetics at the electrode in biosensor cell performance were identified. Saranya et al. [14] discussed a mathematical model of the enzymatic glucose membrane less fuel cell with direct electron transfer.

The analytical results of an enzymatic fuel cell were used, while developing fuel cell, to estimate its various kinetic parameters to attain the highest power value. The influence of the reaction-diffusion parameters and thickness of the enzyme layer on the concentration of hydrogen ions, glucose and enzymes were discussed. The model revealed that increasing the rates of hydrogen ions leads to the higher value of current, voltage and power.

Kirthiga et al. [15] presented a theoretical model of osmium redox polymer mediated glucose oxidase enzyme electrodes. This model is based on a system of three coupled nonlinear reaction-diffusion equations for biochemical reactions occurring in the biofuel cells that describes the oxidized mediator, oxygen and substrate (Glucose) within the biofuel cell. New analytical expressions for the concentration of oxidized mediator, oxygen and substrate and the corresponding current-potential response were derived for all the values of reaction- diffusion parameters. The current-potential response in osmium redox polymer mediated glucose oxidase enzyme electrodes was also discussed. The effect of the diffusion coefficient of a mediator thickens of the film, turnover rate of Glucose Oxidase and Michaelis-Menten constant on the current-potential curve was also reported.

Table 1: Recent contributions to the theoretical modelling of the signal of enzymatic electrodes/ biofuel cells.

| Ref. | Experimental techniques | Enzymatic scheme | Nonlinear Diff. Eqns. with initial/ boundary conditions | Expressions for Concentration and current |
|------|--|---|--|--|
| [2] | Chronoamperometry, normal pulse voltammetry, Steady state voltammetry and limiting current | $S + E_{OX} \leftrightarrow ES \rightarrow P + E_{red}$ $M_{OM} + E_{red} \rightarrow M_{red} + E_{OX}$ $M_{red} \leftrightarrow M_{OX} + ne^{-}S$ | $\frac{\partial M_{OX}(x,t)}{\partial t} = D_M \frac{\partial^2 M_{OX}(x,t)}{\partial x^2} - \frac{(n_s/n_M)K_{cat}[E]}{1+K_M/[M_{OX}]}$ $M_{OX}(X,0) = 0$ $M_{OX}(x=0) = \frac{h_M[M]}{1+h_M} = a[M]$ $M_{OX}(\infty,t) = 0$ | $\frac{M_{OX}(x,t)}{[M]} = \frac{a \cosh\left(\frac{j}{\sqrt{D_M}}[x-l]\right)}{\cosh\left(\frac{j}{\sqrt{D_M}}[-l]\right)} + \frac{2aD_M}{l} \sum_{n=1}^{\infty} \frac{(-1)^{n+1} r e^{-bt}}{b} \cos(r[x-l])$ $\frac{1}{h_M FA} = a[M] D_M \left\{ \frac{j}{D_M} \tanh\left(\frac{j}{\sqrt{D_M}}[-l]\right) + \frac{2D_M}{l} \sum_{n=1}^{\infty} \frac{r^2 \exp(-bt)}{b} \right\}$ |
| [4] | Chronoamperometry, Normal pulse voltammetry, Steady state voltammetry and limiting current | $S + E_{red} \leftrightarrow ES \rightarrow P + E_{OX}$ $M_{red} + E_{OX} \leftrightarrow EM \rightarrow M_{OX} + E_{red}$ $M_{red} + e^{-} \rightarrow M_{OX}$ | $\frac{\partial [Q]}{\partial t} = D_p \frac{\partial^2 [Q]}{\partial x^2} - \frac{C_E^0}{K_1[S] + \frac{1}{K_{1,2}} + \frac{1}{K_{2,2}} + \frac{1}{K_2[Q]}}$ $\frac{\partial [S]}{\partial t} = D_s \frac{\partial^2 [S]}{\partial x^2} - \frac{C_E^0}{K_1[S] + \frac{1}{K_{1,2}} + \frac{1}{K_{2,2}} + \frac{1}{K_2[Q]}}$ $t = 0, \text{ and } x = \infty, x \geq 0, [Q] = 0, [S] = C_s^0$ $x = 0, t \geq 0: [Q] = \frac{C_p^0}{1 + \exp\left[\frac{F}{RT}(E - E_{PQ}^0)\right]}, \frac{\partial [S]}{\partial x} = 0$ $x = \infty, \frac{\partial [Q]}{\partial x} = 0$ | $q(t) = \frac{\exp\left(-\frac{1+e^{-x}}{1+e^{-x}+s}z\right)}{2(1+e^{-x})} \operatorname{erf}\left[\frac{2\sqrt{\frac{1+e^{-x}}{1+e^{-x}+s}}t-z}{2\sqrt{t}}\right] + \exp\left(-\frac{1+e^{-x}}{1+e^{-x}+s}z\right) \operatorname{erfc}\left[\frac{2\sqrt{\frac{1+e^{-x}}{1+e^{-x}+s}}t+z}{2\sqrt{t}}\right] + 1$ $j^c = \frac{\frac{\sqrt{(1+e^{-x})}}{1+e^{-x}+s} \operatorname{erf}\left(\frac{\sqrt{(1+e^{-x})}}{1+e^{-x}+s}\right) + \exp\left(-\frac{(1+e^{-x})}{1+e^{-x}+s}\right)}{(1+e^{-x})\sqrt{pt}}$ |

| | | | | |
|-----|--|--|---|--|
| [5] | Steady state voltammetry and limiting current | $S + E_{red} \xrightleftharpoons[k_{-1}]{k_1} ES \xrightarrow{k_2} P + E_{ox}$ $M_{red} + E_{ox} \xrightleftharpoons[k_{-in}]{k_{in}} EM \xrightarrow{k_{ox}} M_{ox} + E_{red}$ $M_{ox} \leftrightarrow M_{red} + e^-$ | $D_m \frac{d^2 C_m}{dx^2} = \frac{k_{cat} K_m^{-1} c_e c_m c_s}{c_m K_m^{-1} (c_s + K_s) + c_s}$ $D_s \frac{d^2 C_s}{dx^2} = \frac{k_{cat} K_m^{-1} c_e c_m c_s}{4(c_m K_m^{-1} (c_s + K_s) + c_s)}$ $x = 0 : \frac{dc_s}{dx} = 0$ $x = j : c_s = c_s^0, \frac{dc_m}{dx} = 0$ | $c_m(x) = \frac{c_m^0 e^{-j/V_N}}{2 \cosh(V_N)} - \frac{k_{cat} c_e}{D_m K_m} \left(\frac{1}{K_m} + \frac{K_s}{K_m c_s^0} + \frac{2 \cosh(V_N)}{c_m^0 e^{-j/V_N}} \right)^{-1} \times \left(\frac{nFD_j c_m^0}{2i_0 \cosh(V_N)} + jx - \frac{x^2}{2} \right)$ $c_s(x) = c_s^0 - \frac{k_{cat} c_e}{8D_s K_m} \left(\frac{1}{K_m} + \frac{K_s}{K_m c_s^0} + \frac{2 \cosh(V_N)}{c_m^0 e^{-j/V_N}} \right)^{-1} \times (j^2 - x^2)$ $i = -\frac{nFk_{cat} c_e j}{K_m} \left(\frac{1}{K_m} + \frac{K_s}{K_m c_s^0} + \frac{2 \cosh(V_N)}{c_m^0 e^{-j/V_N}} \right)^{-1}$ |
| [7] | Chronoamperometry, Normal pulse voltammetry, Steady state voltammetry and limiting current | $S + E_{ox} \xrightleftharpoons[k_{-1}]{k_1} ES \rightarrow P + E_{red}$ $2M_{ox} + E_{red} \xrightarrow{k_{red}} 2M_{red} + E_{ox}$ $M_{red} \leftrightarrow M_{ox} + e^-$ | $\frac{\partial C_s(x,t)}{\partial t} = D_s \frac{\partial^2 C_s(x,t)}{\partial x^2} - n_{Enz}$ $\frac{\partial C_{MR}(x,t)}{\partial t} = D_M \frac{\partial^2 C_{MR}(x,t)}{\partial x^2} - 2n_{Enz}$ $n_{Enz} = \frac{K_{cat} C_{Enz}}{1 + K_s / C_s + M / C_{MO}}$ $t = 0, C_s = 0; C_{MR} = C_{MT} - M_0^*$ $x = 0, \frac{\partial C_s}{\partial x} = 0; C_{MR} = C_{MT} \left[1 + \exp\left(\frac{nF}{RT}(E - E_{med}^0)\right) \right]^{-1}$ $x = l, C_s = C_{sb}; \frac{\partial C_{MR}}{\partial x} = 0$ | $\frac{C_s(x,t)}{C_{sb}} = \frac{\cosh(\sqrt{j_1} x)}{\cosh(\sqrt{j_1} l)} + \frac{2D_s}{l} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{M_1}{N_1} e^{-N_1 t} \cos(M_1 x)$ $\frac{C_{MO}(x,t)}{C_{MT}} = \frac{\cosh(\sqrt{j_2}(x-l))}{\cosh(\sqrt{j_2} l)} + \frac{2}{l} \sum_{n=1}^{\infty} (-1)^{n+1} \frac{e^{-N_2 t}}{M_1 N_2} \left[dM_1^2 D_M - \frac{M_0^*}{C_{MT}} \right] \cos(M_1(x-l))$ $i = nFA_p D_M \left\{ C_{MT} d\sqrt{j_2} \tanh(\sqrt{j_2} l) - \frac{2C_{MT}}{l} \sum_{n=1}^{\infty} (-1)^{2n-1} e^{-N_2 t} \left[\frac{dM_1^2 D_M}{N_2} - \frac{M_0^*}{C_{MT}} \right] \right\}$ |

3. Numerical Solution of Biofuel Cell

A mathematical model considering reaction and diffusion processes in biofuel cells, verified the effectiveness of a high-surface-area biofuel cell electrode with a thin, grafted redox polymer layer [16]. The response of a glucose dehydrogenase-based biosensor following a synergistic conversion scheme including an enzyme layer together with a dialysis membrane was discussed by Ašeris et al. [17]. The mathematical modeling of enzyme inhibition is discussed numerically by Achi et al. [18]. Porous enzymatic electrodes following DET [19] and MET [20] mechanisms have also been simulated. The models pointed out that the major limitation was mass transfer limitation. Enzyme utilization in both systems were very low.

Theoretical and numerical simulations of diffusion and kinetics in amperometric immobilized enzyme electrodes for redox mediator entrapped within the film using the relaxation method was investigated by Bartlett et al. [21]. Abukhaled and Khuri [22] obtained a semi-analytical solution of amperometric enzymatic reactions. The method was based on constructing a Green's function and employ a fixed point iterative scheme [23, 24]. Barton [25] recently describes some simple modeling approaches for enzymatic fuel cells that are applicable to many other systems, including biosensors, biofuel cell, and bioreactors. Gallaway and Barton et al. [26] discussed the kinetics of redox polymer-mediated enzyme electrodes. Kjeang et al.

[27] proposed a microstructured enzymatic biofuel cell architecture for consecutive chemical reactions. Species transport coupled with laminar flow and Michaelis–Menten kinetics was studied numerically. Microfluidic biofuel cells exploit the lack of active mixing at microscale dimensions to eliminate the use of proton exchange membranes which is separate anolyte and catholyte streams. Simulation of this system, by solving conservation equations (flow, species transport), reveals that oxygen availability limits the performance of the cathode [28].

4. Conclusion

Most mathematical and theoretical models of enzymatic biofuel cells are based on nonlinear reaction-diffusion differential equations containing nonlinear terms related to the kinetics of the enzyme reaction. Various novel and advanced analytical methods such as HPM, HAM, ADM, etc. and numerical methods have been employed to obtain approximate and numerical solutions under steady and non-steady state conditions. Reliable analytical methods are very useful for the analysis of various parameters like the thickness of the electrode, structure of the enzymatic film, the loading of the different species, the diffusivity of the mediator, etc.

In conclusion, enzymatic biofuel cells have made enormous strides in power output and stability since their inception, but more theoretical and simulation work is still needed to make them a more technically and commercially viable option for powering wearable, implantable, and portable devices.

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